Signature: ____________________

Problem:	1	2	3	4	5	Total
Score:						

Each problem is worth 10 points.

1. A group of 11 students were surveyed and asked how many phone calls they make per day. Their responses were

$$3, 7, 10, 7, 5, 11, 10, 6, 8, 4, 6\,.$$

Find the mean and standard deviation of these data.

$\overline{x} =$ ____________ $s =$ ____________

$$\overline{x} = \frac{1}{11}\sum x_i = \frac{77}{11} = \boxed{7}$$

$$s^2 = \frac{1}{10}\sum (x_i - \overline{x})^2 = \frac{66}{10} = 6.6, \text{ so}$$

$$s = \sqrt{6.6} = \boxed{2.569} \text{ to three decimal places.}$$

2. According to government reports, the heights of two-year-old boys is <u>normally</u> distributed with a mean of 34.5 inches and a standard deviation of 1.4 inches.

(a) If a two-year-old boy is chosen at random, what is the probability that he will be between 32.5 and 36.5 inches tall?

$$\text{normalcdf}(32.5, 36.5, 34.5, 1.4) = \boxed{0.84687}\,.$$

(b) Only the tallest 4% of two-year-old boys are treated like royalty by their teachers. What is the height requirement for special treatment?

$$\text{invNorm}(0.96, 34.5, 1.4) = 36.95096\,,$$

so $\boxed{\text{at least } 36.951 \text{ inches.}}$

3. Elaine deposits \$8000 in a savings account paying 3% annual interest compounded monthly. She thereafter makes deposits of \$500 at the end of each month.

(a) Write down the difference equation describing the balance in the account after n months.

$$\boxed{y_0 = 8000}$$

$$\boxed{y_n \quad = \quad (1 + \tfrac{.03}{12})y_{n-1} + 500 \quad \text{or} \quad (1.0025)y_{n-1} + 500}$$

Seriously, that's all I wanted.

(b) Make a table giving the balance in the account after 3, 6, 9, and 12 months. (Be sure to give two decimal points of accuracy.)

months	balance
3	9563.90
6	11139.57
9	12727.07
12	14326.52

4. Jane takes out a 30-year mortgage of $1,200,000 at 8% annual interest compounded monthly.

(a) What is her monthly payment?

This is a decreasing annuity; we know the present value, and want to know the rent. Use the handy-dandy formula:

$$P = \frac{(1+i)^n - 1}{i(1+i)^n} R.$$

Here $P = 1200000$, $i = r/m = .08/12 = 0.00666...$, and $n = 30 \times 12 = 360$. So

$$1200000 = \frac{(1+\frac{.08}{12})^{360} - 1}{\frac{.08}{12}(1+\frac{.08}{12})^{360}} R$$

$$1200000 = 136.2835R$$

$$\boxed{R = \$8805.17}$$

(b) What is the balance remaining on the loan after 20 years?

The question is asking, in other words, what is the present value *20 years from now* of the decreasing annuity with the same rent and interest rate as in part (a). At that point, there are 10 years (120 months) left on the loan, so we use the formula

$$P = \frac{(1+i)^n - 1}{i(1+i)^n} R$$

with P the unknown, $i = 0.00666...$ as above, $R = 8805.17$, and $n = 120$. This gives

$$P = \frac{(1+\frac{.08}{12})^{120} - 1}{\frac{.08}{12}(1+\frac{.08}{12})^{120}} 8805.17$$

$$= (82.42148)(8805.17)$$

$$= \boxed{\$725{,}735.15}$$

5. At the end of each week for 52 weeks, I put \$100 into a savings account paying 5% annual interest, compounded weekly. I then spend 24 weeks travelling, withdrawing a fixed amount each week for my expenses. How much should my withdrawals be? Note that the account continues to accrue interest the whole time. (Hint: divide the problem into two parts.)

The first part is an increasing annuity. We want to find the future value after 52 weeks of a \$100 annuity invested at 5% compounded weekly. We use

$$F = \frac{(1+i)^n - 1}{i} R$$

with $i = \frac{.05}{52}$, $n = 52$, and $R = 100$. This gives

$$F = \frac{(1+\frac{.05}{52})^{52} - 1}{\frac{.05}{52}} 100$$

$$= (53.2957)(100)$$

$$= 5329.57$$

This is what we've saved after 52 weeks. Note that we deposited \$5200, and earned about \$130 in interest (any answer less than \$5200 wouldn't really make sense).

For the second part, we want to take the answer from the first part and construct a decreasing annuity that runs out after exactly 24 weeks. This annuity will have present value equal to our answer from above, \$5329.17, and we want to know the rent. The interest rate is the same as it was before.

$$P = \frac{(1+i)^n - 1}{i(1+i)^n} R$$

$$5329.17 = \frac{(1+\frac{.05}{52})^{24} - 1}{\frac{.05}{52}(1+\frac{.05}{52})^{24}} R$$

$$5329.17 = (23.7139)R$$

$$R = \boxed{\$224.74}$$