Signature: ______________________________

Problem:	1	2	3	4	5	Total
Score:						
out of:	6	11	8	13	12	50

1. (6 points) Put the following system of equations in matrix form (not an equation, just a single matrix).

$$\begin{cases} -2x - 3y + 2z = -2 \\ x + y = 3 \\ -x - 3y + 2z = 1 \end{cases}$$

Solution: $\left[\begin{array}{ccc|c} -2 & -3 & 2 & -2 \\ 1 & 1 & 0 & 3 \\ -1 & -3 & 2 & 1 \end{array}\right]$

2. A 600-seat movie theater charges \$6.50 for adults and \$2.50 for children. If the theater is full and \$3184 is collected, how many adults and how many children are in the audience?

   (a) (1 point) Define your variables:

   $$\begin{cases} x = \underline{\text{the number of adults in the audience}} \\ y = \underline{\text{the number of children in the audience}} \end{cases}$$

   (b) (4 points) Set up the system of linear equations describing the problem.

   Solution: $\begin{cases} x + y = 600 \\ 6.5x + 2.5y = 3184 \end{cases}$

(c) (5 points) Solve your system of equations from (b).

Solution:

$$\begin{cases} x + y = 600 \\ 6.5x + 2.5y = 3184 \end{cases} \longrightarrow \begin{cases} x + y = 600 \\ -4y = -716 \end{cases}$$

$$\longrightarrow \begin{cases} x + y = 600 \\ y = 179 \end{cases}$$

$$\longrightarrow \begin{cases} x = 421 \\ y = 179 \end{cases}$$

(d) (1 point) Answer the problem in a complete sentence.

There are 421 adults and 179 children in the audience.

3. Perform the indicated matrix calculations.

(a) (4 points)

$$\begin{bmatrix} 3 & -1 \\ 2 & 0 \\ 1 & 5 \end{bmatrix} - \begin{bmatrix} 1 & -5 \\ 3 & 2 \\ -3 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & 4 \\ -1 & -2 \\ 4 & 5 \end{bmatrix}}$$

(b) (4 points)

$$\begin{bmatrix} 3 & 1 & 2 \\ -1 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} 7 & -1 \\ 5 & 4 \\ -6 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 14 & 1 \\ -10 & 1 \end{bmatrix}}$$

4. Consider the system of linear equations

$$\begin{cases} 4x & + & 3y & = -6 \\ 2x & + & 4y & = 2 \end{cases}$$

(a) (4 points) Transform the system of equations into a matrix equation of the form $AX = B$:

$$A = \begin{bmatrix} 4 & 3 \\ 2 & 4 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$B = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$$

$AX = B$:

$$\begin{bmatrix} 4 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$$

(b) (2 points) Find Δ (the determinant of A).

$$\Delta = (4)(4) - (2)(3) = \boxed{10}$$

(c) (3 points) Find A^{-1}.

$$A^{-1} = \begin{bmatrix} \frac{4}{10} & \frac{-3}{10} \\ \frac{-2}{10} & \frac{4}{10} \end{bmatrix} = \boxed{\begin{bmatrix} .4 & -.3 \\ -.2 & .4 \end{bmatrix}}$$

(d) (3 points) Use your answer from (c) to solve for X.

$$X = A^{-1}B = \begin{bmatrix} .4 & -.3 \\ -.2 & .4 \end{bmatrix} \begin{bmatrix} -6 \\ 2 \end{bmatrix} = \boxed{\begin{bmatrix} -3 \\ 2 \end{bmatrix}}$$

(e) (1 point) What is the solution of the original system of equations?

$$\begin{cases} x = \boxed{\text{-3}} \\ y = \boxed{2} \end{cases}$$

5. Suppose an economy consists of three sectors: Coffee, Beer, and Music. To produce \$1 worth of coffee requires \$.30 worth of coffee, \$.20 worth of beer, and \$.30 worth of music; to produce \$1 of beer you need \$.30 worth of coffee, \$.10 worth of beer, and \$.50 worth of music; to produce \$1 worth of music requires \$.30 worth of coffee and \$.40 worth of beer (and no music). How much should each industry produce if the public demands \$20 million worth of coffee, \$12 million worth of beer, and \$34 million worth of music?

(a) (1 point) Define your variables:

$$\begin{cases} x = \text{the amount of coffee produced} \\ y = \text{the amount of beer produced} \\ z = \text{the amount of music produced} \end{cases}$$

(b) (4 points) Set up a matrix equation of the form $(I - A)X = D$ to describe this input-output problem.

$$A = \begin{bmatrix} .3 & .3 & .3 \\ .2 & .1 & .4 \\ .3 & .5 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$D = \begin{bmatrix} 20 \\ 12 \\ 34 \end{bmatrix}$$

$(I - A)X = D$:

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} .3 & .3 & .3 \\ .2 & .1 & .4 \\ .3 & .5 & 0 \end{bmatrix} \right) \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 12 \\ 34 \end{bmatrix}$$

$$\begin{bmatrix} .7 & -.3 & -.3 \\ -.2 & .9 & -.4 \\ -.3 & -.5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 12 \\ 34 \end{bmatrix}$$

(c) (1 point) A is called the input-output (or technology) matrix, and

D is called the final demand matrix.

(d) (2 points) Find $(I - A)^{-1}$. (Use your calculator for this part.)

$$(I - A)^{-1} = \begin{bmatrix} 2.4735 & 1.5901 & 1.3781 \\ 1.1307 & 2.1555 & 1.2014 \\ 1.3074 & 1.5548 & 2.0141 \end{bmatrix}$$

(to four decimal points, just for fun.)

(e) (3 points) Use your answer to (d) to solve for X. (You'll want to use your calculator.)

$$\begin{aligned} X &= (I - A)^{-1}D \\ &= \begin{bmatrix} 2.4735 & 1.5901 & 1.3781 \\ 1.1307 & 2.1555 & 1.2014 \\ 1.3074 & 1.5548 & 2.0141 \end{bmatrix} \begin{bmatrix} 20 \\ 12 \\ 34 \end{bmatrix} \\ &= \boxed{\begin{bmatrix} 115.40636 \\ 89.3286 \\ 113.2862 \end{bmatrix}} \end{aligned}$$

(f) (1 point) Answer the problem in a complete sentence.

The Coffee industry should produce \$115.406 million worth of coffee, the Beer industry should produce \$89.328 million worth of beer, and the Music industry should produce \$113.286 million worth of music.