Definite Integral / Sums

Strategy: Given f(x) on an interval [a,b] and n = # of subintervals.

(1) Partition the interval.
- $\Delta x = \dfrac{b-a}{n}$
- $x_0 = a$, $x_1 = a + \Delta x$, $x_2 = a + 2\Delta x$,..., $x_i = a + i\Delta x$,...,$x_n = b$

(2) Draw rectangles and add area
- With left endpoints: $A = \sum_{i=0}^{n-1} f(x_i)\Delta x$
- With right endpoints: $A = \sum_{i=1}^{n} f(x_i)\Delta x$

(3) To find exact sum, take $\lim_{n\to\infty}$

Problems:

(1) Given $f(x) = x^2 - 3$ over the interval [2, 3]
   (a) Estimate the area under the curve using right endpoints and n=5.
   (b) Estimate the area under the curve using left endpoints and n=5.
   (c) Estimate the area under the curve using middle points and n=5.
   (d) Find the exact area under the curve.

(2) Given $f(x) = x + 3$ over the interval [-1,1]
   (a) Estimate the area under the curve using right endpoints and n=6.
   (b) Estimate the area under the curve using left endpoints and n=6.
   (c) Estimate the area under the curve using middle points and n=6.
   (d) Find the exact area under the curve.

(3) Find $\int_0^2 (x+1)dx$ using the definition.

(4) Find $\int_0^2 (x^2+1)dx$ using the definition.

(5) Find $\int_{-2}^1 (3x^2+2)dx$ using the definition.