MAT 397 — FALL 2007 — EXAM I REVIEW

Note that this is not meant to be a comprehensive review. It is intended to remind you the sorts of things we've worked on, and to give you a chance to ask questions about typical problems.

Questions **1.** through **12.** refer to the vectors $\vec{u} = \langle 3, 1, -2 \rangle$ and $\vec{v} = \langle 0, -2, 3 \rangle$.

1. Find $6\vec{u} - 4\vec{v}$.

2. Find $|\vec{u}|$ and a unit vector in the direction opposite to $\vec{u}$.

3. Find $\vec{u} \cdot \vec{v}$.

4. Find $\vec{u} \times \vec{v}$.

5. Find the cosine and sine of the angle from $\vec{u}$ to $\vec{v}$.

6. Find the direction cosines of $\vec{u}$.

7. Find the scalar projection of $\vec{u}$ onto $\vec{v}$ and the vector projection of $\vec{v}$ onto $\vec{u}$.

8. Find two vectors orthogonal to $\vec{v}$.

9. Find the area of the parallelogram with vertices at $(0,0,0)$, $(3,1,-2)$, and $(0,-2,3)$ (note that this is only three vertices – how to find the fourth? do you need it?).

10. Find an equation for the circle centered at the tip of $\vec{u}$ with radius $|\vec{v}|$. Put the equation in standard form.

11. Find parametric and symmetric equations for the line through the point $(4,4,4)$ in the direction of $\vec{u}$. Find the coordinates of the points where that line intersects the three coordinate planes. Find the equation of the plane containing that line and the origin.

12. Find an equation for the plane containing the point $(1,1,1)$ and with normal vector $\vec{u}$. Find its intercepts. Find the distance from the point $(0,-2,3)$ to that plane.

13. Sketch the traces of the quadric surface defined by $x^2 + y^2 - z^2 = 1$ in the planes $x = k$, $y = k$, $z = k$ for a few values of k. Sketch the surface.

14. For each of the following equations in cylindrical coordinates, describe the graph and convert to Cartesian coordinates.

(a) $r = 0$

(b) $r^2 + z^2 = 25$

(c) $r^2 - z^2 = 1$

(d) $z = -1$

(e) $r = -2\sec\theta$

(f) $z = 2r^2$

15. Same question, but these equations are in spherical coordinates.

(a) $\rho = 2$

(b) $\phi = \pi/4$ (don't convert)

(c) $\theta = \pi/6$ (don't convert)

(d) $\rho = 8\sec\theta$

16. Find the length of the curve traced out by $\vec{r}(t) = 2\cos t\ \vec{i} + 2\sin t\ \vec{j} + t^2\ \vec{k}$ from $t = 0$ to $t = \pi/4$. Find vectors tangent to the curve at those two t-values.

17. S'pose a point is moving in the plane with position vector given as a function of time by $\vec{r}(t)$. If the velocity of the point is given by $\vec{v}(t) = 3t^2\ \vec{i} - \sin(3t)\ \vec{j}$, and $\vec{r}(0) = 2\ \vec{i} + \frac{4}{3}\ \vec{j}$, find $\vec{r}(t)$.

18. If a particle moves in space with position vector given by $\vec{r}(t) = \cos t\ \vec{i} + \frac{1}{2}t^2\ \vec{j} - t\ \vec{k}$, find $\vec{v}(t)$, $\vec{a}(t)$, $\mathbb{T}(t)$, $\mathbb{N}(t)$, and $\mathbb{B}(0)$. (Find $\mathbb{T}$ and $\mathbb{N}$ in general, then plug in $t = 0$ before finding $\mathbb{B}$.) Find $|\mathbb{B}(t)|$ (you don't need to find $\mathbb{B}(t)$ to do this).

19. A javelin leaves the thrower's hand 2 meters above the ground at a 45° angle and at 30 m/s. How high does it go? How far?