MAT 397 — SPRING 2006 — EXAM II REVIEW

Note that this is not meant to be a comprehensive review. It is intended to remind you the sorts of things we've worked on, and to give you a chance to ask questions about typical problems.

(1) A javelin leaves the thrower's hand 2 meters above the ground at a 45° angle and at 30 m/s. How high does it go? How far?

(2) Find the length of the curve traced out by $\vec{r}(t) = 2\cos t\ \vec{i} + 2\sin t\ \vec{j} + t^2\ \vec{k}$ from $t = 0$ to $t = \pi/4$.

(3) S'pose a point is moving in the plane with position vector given as a function of time by $\vec{r}(t)$. If the velocity of the point is given by $\vec{v}(t) = 3t^2\ \vec{i} - \sin(3t)\ \vec{j}$, and $\vec{r}(0) = 2\ \vec{i} + \frac{4}{3}\ \vec{j}$, find $\vec{r}(t)$.

(4) If a particle moves in space with position vector given by $\vec{r}(t) = \cos t\ \vec{i} + \frac{1}{2}t^2\ \vec{j} - t\ \vec{k}$, find $\vec{v}(t)$, $\vec{a}(t)$, $\mathbb{T}(t)$, $\mathbb{N}(t)$, and $\mathbb{B}(0)$. (Find $\mathbb{T}$ and $\mathbb{N}$ in general, then plug in $t = 0$ before finding $\mathbb{B}$.)

(5) Let $f(x, y) = \frac{1}{3}\sqrt{36 - 9x^2 - 4y^2}$. What is the domain of f? Sketch 3 level curves of the graph of this function. Be sure to indicate the scale and label each curve properly.

(6) For the following problems, find the limit or explain clearly why it does not exist.
   - (a) $\lim_{(x,y)\to(1,\pi/2)} \frac{2+x}{x+\cos y}$
   - (b) $\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$
   - (c) $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$
   - (d) $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$

(7) Find the indicated partial derivatives.
   - (a) $\frac{\partial h}{\partial x}$ for $h(x, y) = (3x^2 + y^2)^{-1/2}$
   - (b) f_y for $f(x, y) = \frac{\sin(e^{xy})}{x^2 y}$
   - (c) g_{xy} for $g(x, y, z) = x^2 y e^{xy-z} + \ln(5y - 2z)$
   - (d) All four second partial derivatives for $f(x, y) = x + \frac{y}{x}$. Is $f_{xy} = f_{yx}$? Why or why not?

(8) Find the slope of the tangent line to the curve of intersection of the surface defined by $z = 4 - x^2 - 2y^2$ and the plane $x = 1$ at the point $(1, 1, 1)$.

(9) Use the definition of differentiability to show that $f(x, y) = 2x + y^2$ is differentiable.

(10) Find an equation for the tangent plane to the graph of $z = xe^{-2y} + y(x^2 - y^2)$ at the point $(1, 0, 1)$.

(11) Find an equation for the plane tangent to $z = \ln(x^2 + y^2)$ at $(0, 1, 0)$. Use a linear approximation to estimate the value of $\ln(1.01)$ in two ways: by taking $x = 0, y = 1.005$, and by taking $x = .1, y = 1$.