MAT 397 — SPRING 2006 — EXAM III REVIEW

Note: This is not meant to be a comprehensive review. It is intended to remind you the typical sorts of problems that we've considered. I've given some below, and in places where your textbook has decent problems, I've just pointed at them. There are many others on the syllabus.

Exam III covers two general topics: Optimization of Multivariate Functions, and Integration in Two and Three Variables.

§14.6 The Gradient and Directional Derivatives

Calculating the gradient of a function should be straightforward by now, and you should also brush up on how to obtain the directional derivative in the direction of a given unit vector. See #7–15. We also interpret the gradient as giving the direction of fastest increase of the function (but be careful if the function gives depth!).

§14.7 Maximum and Minimum Values

You should be able to classify all critical points of a function, whether on all of the xy-plane or on a closed bounded set. It's very important to state your answer clearly and carefully, something like "$f(x, y)$ attains its maximum value at $(3, -\pi)$, and that maximum value is $\sqrt{2}$."

- Find and classify the critical points of $f(x, y) = x^3y + 12x^3 - 8y$, $g(x, y) = xy(1 - x - y)$, and $h(x, y) = x \sin y$.
- Find the absolute maximum value and absolute minimum value of $f(x, y) = x^2 + y^2$ on the rectangle $S = \{(x, y) \ : \ -1 \leq x \leq 3, -1 \leq y \leq 1\}$.
- (the dreaded distance problem) Find the point on the plane defined by $x+y-z = 1$ that is closest to the point $(2, 1, -1)$. What is the distance?

§14.8 Lagrange Multipliers

These can get hairy. Be sure you understand several techniques for solving the systems of equations that you get.

- Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the unit circle. (This can be done without Lagrange multipliers – you should do it both ways to be sure you get the same answer.)
- Same question, $f(x, y) = y^2 - x^2$ on the ellipse $x^2 + 4y^2 = 4$.

§15.1 Double Integrals over Rectangles

This is mostly a conceptual section, without many good problems. You should look back at #11–13, though.

§15.2 Iterated Integrals

This is where things get good. Some fine problems on computing iterated integrals are #1–20, 27, 29. Let me know if you want more.

§15.3 Double Integrals over General Regions

The most important thing in this section is being fluent in translating between (a) descriptions of regions in the plane, (b) sketches of those regions, and (c) limits of integration that cover those regions. Problems #37–42 are good for this. Of course, computing the integrals is important too, so you should be able to work problems like #1–18.

- Evaluate $\iint_D xy\,dA$ where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = x + 1$.

§15.4 Double Integrals in Polar Coordinates

This section is again mostly computational. The most important thing is not to forget the r when translating an integral from Cartesian coordinates to polar coordinates. Problems #7, 13, 21, 25, 27 are classic. Don't worry too much about graphing difficult polar functions.

§15.5 Applications

You need to know how to find the mass of a lamina and the coordinates of its center of mass. Don't worry about the rest of this section (moment of inertia, expected value of a probability distribution, etc.). Problems #1 and 5 are classics here.

§15.7 Triple Integrals

From a computational point of view, triple iterated integrals are just like the double iterated ones from 15.2 and 15.3, only 1.5 times as long. From a conceptual point of view, the real difficulty is in visualizing and describing regions of integration in space. Try #25 and 31 for practice with this, and #9, 11, 17 for computation practice.

- Evaluate the triple integral $\iiint_E x^2y^2\,dV$, where E is the solid region under the plane $z = 2 + 2x + 2y$ and above the region in the xy-plane bounded by the curves $y = \sqrt{x}$, $y = 0$, and $x = 1$.