# MAT 514 — FALL 2007 — FINAL EXAM REVIEW

This review sheet covers (most of) Chapters 5 and 6 of the text. (Since you already have review sheets and exams covering the rest of the course, there's no need to reinvent that wheel.) As always, it's intended to point out the main topics we covered, remind you of the sorts of problems we discussed, and give you a little more practice with routine problems. Here is a reminder about the sections and topics under consideration. I'll also throw a few basic Laplace Transform problems in below, so you can get all the practice you need with them.

(1) Sections 6.3, 6.4, and 6.5: Solving DEs with discontinous forcing functions (step functions, impulse functions) via Laplace transforms
(2) Section 6.6: The convolution product (not on the final)
(3) Sections 5.2 and 5.3: Solving DEs with power series near an ordinary point
(4) Sections 5.4 and 5.5: Solving Euler's equation near a regular singular point

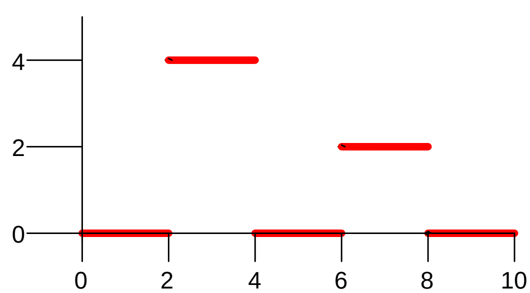
**Practice Problems:**

**1.** Write the following function in terms of step functions.

$$h(t) = \begin{cases} 1 & t < 0 \\ 1 - t^2 & 0 \leq t < 1 \\ t + 1 & 1 \leq t < 2 \\ 2 & t \geq 2 \end{cases}$$

**2.** Use the definition of the Laplace transform as an indefinite integral to evaluate $\mathscr{L}(u_1(t)e^{-t})$.

**3.** Find the Laplace transform of this function:



**4.** Find the Laplace transform of the solution to the following initial value problem. Do not bother to find the solution of the initial value problem.

$$4y'' - 2y' + 3y = 1 + 3\cos t, \qquad y(0) = -2, y'(0) = 1$$

**5.** Find the Laplace transform of $f(t)$.

(a) $\sin 3t + \cos 3t$

(b) $e^t(1 + \cos 2t)$

(c) $t^2 - u_1(t)(t^2 - 1)$

(d) $u_2(t)(t+1)$

(e) $t^2\delta(t-1)$

**6.** Find the inverse Laplace transform of $F(s)$.

(a) $\dfrac{s}{(s-1)^2}$

(b) $\dfrac{s}{s^2 - 2s - 3}$

(c) $\dfrac{se^{-s}}{s^2 + 2s + 5}$

**7.** Solve the initial value problem.

(a) $y'' + y = h(t)$, where $h(t) = \begin{cases} t & 0 \le t < \pi \\ \pi & t \ge \pi \end{cases}$, with $y(0) = y'(0) = 0$.

(b) $y'' + y = \delta(t - \pi)$, with $y(0) = y'(0) = 0$.

(c) $y'' + y = h(t)$, where $h(t) = \begin{cases} 4 & 0 \le t < 2 \\ t+2 & t \ge 2 \end{cases}$, with $y(0) = y'(0) = 0$. (This is long!)

(d) $y'' + 4y + 4y = u_2(t)$, $y(0) = 0$, $y'(0) = 2$.

**8.** Find the first six coefficients in a series solution of the following equation about $x_0 = 0$.

$$(x^2+1)y'' - 4xy' + 6y = 0$$

**9.** Find the first three nonzero terms of two linearly independent power series solutions about $x_0 = 1$.

$$y'' + (x+1)y' - 2y = 0$$

**10.** Find the first three nonzero terms of two linearly independent solutions of $xy'' + 2y = 0$. (Note that $x_0 = 0$ is a singular point.)

**11.** Where will a series solution be certain to converge?

(a) $(x-1)(x-2)y'' + xy' + 2y = 0$ about $x_0 = 0$

(b) $(x^2+1)y'' + xy' + 2y = 0$ about $x_0 = 1$

**12.** Find the indicial polynomial, its roots, and the general solution.

(a) $x^2y'' - 3xy' + 13y = 0$

(b) $x^2y'' - 2y = 0$

(c) $x^2y'' + 5xy' + 4y = 0$

(d) $x^2y'' - 7xy' + 16y = 0$

(e) $2x^2y'' + 3xy' - 15y = 0$

(f) $x^2y'' + 3xy' + 4y = 0$