

## MAT 532 — HOMEWORK 6

DUE ON WEDNESDAY, MARCH 28

**Reading.** Read sections 9.3, 9.4, 11.6, and 11.9 carefully.

### Problems to hand in.

1. Find the least squares solution to

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \vec{x} = \begin{bmatrix} 1 \\ -1 \\ 1 \\ 12 \end{bmatrix}$$

2. Find the regression line for the following set of points:  $(-1, 2)$ ,  $(0, 1)$ ,  $(1, -3)$ ,  $(2, -4)$ .
3. Suppose you're told that  $Z$  and  $t$  are related by a function of the form  $Z = a + be^t$ , for some  $a$  and  $b$ . Estimate  $a$  and  $b$  given the following data (that is, use least squares to fit the best equation of this form to the data):  $(t, Z)$ :  $(0, 1)$ ,  $(1, 2)$ ,  $(2, 4)$ .
4. Define

$$f(x) = \begin{cases} -\pi & \text{if } 0 \leq x \leq \pi, \text{ and} \\ \pi & \text{if } \pi < x \leq 2\pi. \end{cases}$$

Find the projection of  $f(x)$  onto the subspace

$$\text{span}\{1, \sin x, \sin 2x, \cos x, \cos 2x\}$$

of  $C[0, 2\pi]$ . (You'll have to split each integral up into two parts.) Use your calculator or a computer to draw the graph of the projection (and hand in a sketch).

5. There are 100,000 women in the city of Syracuse. (For some obscure reason, this number never changes.) Suppose that, each year, 28% of the single women get married, and 18% of the married women get divorced or become single again one way or another. Approximately how many women will be single in the long run?

**Problems *not* to hand in.**

1. Suppose there are three coffeeshop companies in town: Xarbucks, Yarbucks, and Zarbucks. Let's abbreviate their names, shall we? Each year X loses  $\frac{1}{3}$  of its customers to Y, but none to Z, which is inconveniently located. Meanwhile, Y loses just  $\frac{1}{6}$  of its customers to X and  $\frac{1}{3}$  of its customers to Z. Finally, Z always loses  $\frac{1}{3}$  of its customers to X, due to the aforementioned location troubles. Write down the matrix associated to this Markov chain. If Y initially had *all* the customers, how many would it have 2 years later? If the initial distribution was 300 X-customers, 200 Y-customers, and 100 Z-customers, how many customers should Z expect to have in the long run?
  2. #11.6.3
  3. #11.9.5
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**Argument, Bill Parry**

As he cleaned the board,  
chalk-dust rose like parched mist.  
A dry profession, he mused as morosely  
they shuffled settling tier upon tier.

Now, almost half-way through the course,  
(coughs, yawns and automatic writing)  
the theorem is ready.

Moving to the crucial point,  
the sly unconventional twist,  
a quiver springs his voice and breast;

soon the gambit will appear  
opposed to what's expected.  
The ploy will snip one strand  
the entire skein sloughing to the ground.

His head turns sympathetically  
from board to class.  
They copy copiously.  
But two, perhaps three pause and frown,

wonder will this go through,  
questioning this entanglement  
—yet they nod encouragement.  
Then the final crux; the ropes relax and  
fall.

His reward: two smile, maybe three,  
and one is visibly moved.  
Q.E.D., the theorem is proved.

This was his sole intent.  
Leaving the symbols on the board  
he departs with a swagger of achieve-  
ment.