MAT 534 — HOMEWORK 4

DUE ON THURSDAY, 25 FEBRUARY 2009

All these problems are from Chapter 5.

1. #1 and #3, slightly modified: Find the orders of the following permutations.
   (a) (14)
   (b) (14762)
   (c) (124)(357)
   (d) (124)(357869)

2. #2: Write each of the following permutations as a product of disjoint cycles.
   (a) (1235)(413)
   (b) (12)(13)(23)(142)

3. #4: What is the order of the following permutations?

$$\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 5 & 4 & 6 & 3 \end{bmatrix} \qquad \text{and} \qquad \tau = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & \\ 7 & 6 & 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

4. #8: What is the maximum order of any element in A_{10}?

5. #10: Show that a function from a finite set S to itself is one-to-one if and only if it is onto. (This is false if S is infinite.)

6. #13: Prove Theorem 5.6, that the set A_n of even permutations in S_n forms a subgroup.

7. #17: Let

$$\sigma = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 3 & 5 & 4 & 6 \end{bmatrix} \qquad \text{and} \qquad \tau = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{bmatrix}.$$

Compute σ^{-1}, $\tau\sigma$, and $\sigma\tau$.

8. #23: Use Table 5.1 on page 107 (the Cayley table for the group A_4 of even permutations on $\{1,2,3,4\}$) to compute the following.
   (a) the centralizer of $\alpha_3 = (13)(24)$.
   (b) the centralizer of $\alpha_{12} = (124)$.

9. #24: How many elements of order 5 are in S_7?

10. #31: Let G be a group of permutations on a set X. Let $a \in X$ and define stab(a) to be the set of permutations $\sigma \in G$ such that $\sigma(a) = a$. We call stab(a) the *stabilizer* of the element a. Prove that for any a, stab(a) is a subgroup of G.

11. #36: In S_4, find a cyclic subgroup of order 4 and a non-cyclic subgroup of order 4.

Also suggested (but not to hand in): Chapter 5: #16, 18, 25, 37, 38, 41, 48, 54