

MAT 631 — PROBLEM SET 3

Hand in your solution to Problem 1 by 5 pm, Wednesday, Sept 27.
The other problems may be turned in at any time.

1. Prove the *Correspondence Theorem*: Let $\varphi: G \rightarrow G'$ be a surjective homomorphism with kernel N . Then the set of subgroups H' of G' is in bijective correspondence with the set of subgroups H of G that contain N . The correspondence is given by $H \mapsto \varphi(H)$ and $H' \mapsto \varphi^{-1}(H')$. Furthermore, normal subgroups of G' correspond to normal subgroups of G .
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2. Prove that a subgroup of index 2 is necessarily normal.
 3. Classify the groups of order 6 by analyzing three cases: there exists an element of order 6; there is an element of order 3 but none of order 6; and all elements have order 1 or 2. (Explain why these are the only cases you need to consider.)
 4. Prove that the product of two infinite cyclic groups is not cyclic.
 5. Let G be a finite group whose order is a product of two integers, $n = ab$. Let H and K be two subgroups of orders a and b , respectively, and assume that $H \cap K = \{1\}$. Prove that $HK = G$. Is G isomorphic to the product $H \times K$?