

## MAT 631 — PROBLEM SET 9

Hand in your solution to Problem 1 by 5 pm, Monday, November 20. The other problems may be turned in at any time.

1. Classify the groups of order 35.

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2. True or false: the only Abelian simple groups are cyclic of prime order. (Do not assume finiteness.)
3. Prove that a group of order  $2p$ , with  $p$  an odd prime, is either cyclic or dihedral. (There is a tricky step near the end; hints available.)
4. Let  $G$  be a group of order  $n$ , and  $k$  any field. Prove that  $G$  is isomorphic to a subgroup of  $\text{GL}_n(k)$ . (Culture note: a group homomorphism  $G \rightarrow \text{GL}_m(k)$ ,  $m$  any integer, is called a *representation* of  $G$  over  $k$ . In particular,  $m = 1$  gives the characters mentioned on a previous problem set.)