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Name _____

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Exam Score: _____/100

Put a box around your answer.

1. (3 pts each) Mark the following statements True or False. You need not justify your answer.

(a) If $f'(x) > 0$ for all $x \in [a, b]$, then the minimum value of f on $[a, b]$ is attained at a .

(b) If $f''(c) = 0$ then f has an inflection point at $(c, f(c))$.

(c) The graph of the function $f(x) = 3x^6 + 4x^4 + 2x^2$ is concave up everywhere.

(d) A continuous function on a closed interval must attain a maximum on that interval.

(e) If $f'(3) = 0$ and $f''(x) < 0$ for all $x > 3$, then f is decreasing on $(3, \infty)$.

(f) A function can have infinitely many critical points.

(g) The function $f(x) = |x|$ satisfies the hypotheses of the Mean Value Theorem on the interval $[-1, 1]$.

(h) If $f'(c) = f''(c) = 0$, then f has both a local minimum and a local maximum at c .

2. (10 pts) Find the global extrema of the function $f(x) = 1 + 4x^2 - \frac{1}{2}x^4$ on the interval $[-1, 3]$. Your answer should be ordered pairs (x, y) , identified as global maxima or global minima.

3. (10 pts) Find the point(s) on the curve defined by $y = \frac{1}{8}x^2$ closest to the point $(0, 6)$.

4. Let $f(x) = 3x^{5/3} - 5x$.

(a) (4 pts) Find the domain of f .

(b) (4 pts) Find all x -intercepts and y -intercepts. Your answers should be ordered pairs (x, y) .

(c) (5 pts) Find all critical points of f , and classify them as local maxima, local minima, or neither. Your answers should be ordered pairs (x, y) .

(d) (3 pts) On what intervals is $f(x)$ increasing/decreasing?

(e) (5 pts) Find all inflection points $(x, f(x))$ of f .

(f) (3 pts) On what intervals is the graph of f concave up/down?

(g) (8 pts) Sketch a graph of $y = f(x)$, labelling all the points from above.

5. (12 pts) A Whiffleball field is in the shape of an equilateral triangle with a semicircle on top (see the figure). What is the largest possible area of the field if its perimeter is 160 meters? (Hint: the area of an equilateral triangle with side length s is $A = \frac{\sqrt{3}}{4}s^2$.)

6. Let $f(x) = 8\sqrt{x} + 5$ on the interval $[4, 9]$.

(a) (4 pts) Find the average slope of $f(x)$ on this interval.

(b) (4 pts) What does the Mean Value Theorem say?

(c) (4 pts) Find a point c in the open interval $(4, 9)$ such that $f'(c)$ is equal to the average slope.