MATHEMATICS 295

FINAL EXAMINATION – FALL 2000

Print Your Name______________________________________

Signature______________________________________

Print Your Instructor's Name______________________________________

Section #________, Recitation Section #________

Student Identification Number______________________________________

INSTRUCTIONS. This examination has 11 problems and 10 printed pages. (There are 2 additional pages for scrap work.) **Make sure your examination copy is complete before you begin work.**

There are 200 points available on this examination. The point values are indicated for each of 12 problems.

All work for which you seek credit must be written on the printed pages in the appropriate places. The last two scrap pages of this booklet will not be graded! **All answers must be justified.**

Do not write below this line

1.____________ 7.____________

2.____________ 8.____________

3.____________ 9.____________

4.____________ 10.____________

5.____________ 11.____________

6.____________ Total____________

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

1. (8 points each) Evaluate the following limits:

a) $\lim_{x\to 1} \frac{x^2 - 2x + 1}{x^3 - 2x^2 + x}$

b) $\lim_{x\to\infty} \frac{x^4 + x^3}{12x^3 + 128}$

c) $\lim_{x\to 0} \frac{4x}{\sin 2x}$

d) $\lim_{x\to 3+} \frac{|x-3|}{x-3}$

2. (9 points each) Find the derivatives of each of the following functions. *Do not simplif*

a) $f(x) = x^\pi - \dfrac{1}{x^4} + \dfrac{x+1}{x^2+2}$.

b) $f(x) = (\ln(1+x^3))(\sec x)$

c) $f(x) = x^{\cos x}$

d) $f(x) = \left(\dfrac{e^{x^2}}{8+\tan x}\right)^7$

3. (10 points) Find the equation of the tangent line to the curve: $2xy + \pi \sin y = 2\pi$ at the point $(1, \frac{\pi}{2})$.

4. (10 points) Find the linearization of the function $f(x) = \sin(\ln(1 + x))$ at $x = 0$. Use the linearization, or differentials, to estimate $\sin(\ln(1.1))$.

5. (15 points) A balloon is rising vertically above a level, straight road at a constant rate of 2 feet per second. A bicyclist is pedaling along the road at a constant rate of 20 feet per second. Just when the balloon is 100 feet above the ground, the bicyclist passes underneath it. How fast is the distance between the bicycle and the balloon increasing 4 seconds later?

6. (15 points) A rectangle has one vertex at the origin. Two of its sides lie on the positive x and positive y axes. The remaining vertex P is on the part of the graph of $y = \dfrac{1}{x^3+16}$ in the first quadrant. Find the coordinates of P which make the area of the rectangle a maximum. For full credit you must justify your answer using either the first or second derivative test.

7. (5 points each) Consider the following function: $f(x) = \dfrac{x^2 + 2x - 4}{x^2}$.

a) Determine the horizontal asymptote(s) of f or tell why it does not have any.

b) Determine the vertical asymptote(s) of f.

c) Determine the left hand and right hand limits of $f(x)$ at each vertical asymptote.

d) Find $f'(x)$ (the first derivative of f).

e) Determine the intervals where f is decreasing or increasing; find any local maxima and minima of f.

f) The second derivative of f is given by $f''(x) = \dfrac{4(x-6)}{x^4}$. (Do NOT verify this. You will run out of time if you do.) Determine the intervals where f is concave up or down, and, find any inflection points of f.

g) Using the ABOVE, sketch the graph of f showing and LABELING all of its interesting features. NO credit will be given for only copying a picture from your graphing calculator.

8. (7 points) The equation $x^2 = 2^x$ has three solutions: $x = 2$, $x = 4$, and one other. S carefully that the third solution lies in the interval $[-1, 0]$. Explain your reasoning carefully credit will be given for copying pictures from your calculator.

9. (7 points) Find the derivative of

$$f(x) = \int_0^{x^2} e^{\sin t}\, dt.$$

10. (11 points each) Evaluate the following indefinite and definite integrals:

a) $\int_0^1 t^2(t^3-1)^5\,dt$

b) $\int \csc^2 2x \cot 2x\,dx$

11. (11 points) Find the total area of the region between the curve $y = x^3 - 4x$ and th
for $-2 \leq x \leq 0$.