MATHEMATICS 295
FINAL EXAMINATION – SPRING 2002

Print Your Name____

Signature____

Print Your Instructor's Name____

Section #________, Recitation Section #________

Student Identification Number____

INSTRUCTIONS. This examination has 11 problems and 10 printed pages. (There are 2 additional pages for scrap work.) **Make sure your examination copy is complete before you begin work.**

There are 200 points available on this examination. The point values are indicated for each of the 11 problems.

All work for which you seek credit must be written on the printed pages in the appropriate places. The last two scrap pages of this booklet will not be graded! **All answers must be justified. Calculators may be used to check your answers but not to justify them.**

Do not write below this line

1.______________ 7.______________

2.______________ 8.______________

3.______________ 9.______________

4.______________ 10.______________

5.______________ 11.______________

6.______________ Total______________

1. (8 points each) Evaluate the following limits. If a limit does not exist, say so and explain why not:

a) $\lim_{x\to 1} \frac{2x^2 - 4x + 2}{x^3 - 2x^2 + x}$

b) $\lim_{x\to\infty} \frac{x^4 + x^3}{12x^5 + 128}$

c) $\lim_{x\to 0} \frac{2x}{\tan 3x}$

d) $\lim_{x\to 1} \frac{|x - 1|}{x - 1}$

2. (10 points each) Find the derivatives of each of the following functions. **Do not simplify.**

a) $f(x) = x^e - \dfrac{1}{x^9} + \dfrac{\sin x}{x^4 + 5}$.

b) $f(x) = e^{x^2}(\ln(2 + x^2))$

c) $f(x) = \tan(\sin x)$

d) Using logarithmic differentiation, find $f'(x)$ if $f(x) = \dfrac{(x^2 + 1)^{1/3}}{x^{\frac{7}{2}}(7 + \sec x)^4}$.

3. (10 points) Find the equation of the tangent line to the curve: $x^3 y + y^3 x = 30$ at the point $(1, 3)$.

4. (8 points) Use linear approximation (differentials) to approximate $f(x) = x^4 -$ $a = 2$. Use it to estimate $f(x)$ at $x = 2.1$

5. (15 points) A shallow concrete reservior is in the shape of an inverted cone of radius 45 feet and height 6 feet. Water is leaking from the bottom (the vertex) at the rate of 50 cubic feet per minute. How fast is the water level falling when the water is 5 feet deep? The volume V of a cone is given by $V = \frac{1}{3}\pi r^2 h$ where r is the radius of the cone and h is its height.

6. (15 points) A right triangle has one vertex at the origin, its right angle on the positive x-axis, and its third vertex P on the graph of $y = \dfrac{x}{(x+3)^3}$. Find the x coordinate of the point P which causes the triangle to have maximum area. For full credit you must justify your answer using either the first or second derivative test.

7. (5 points each) Consider the following function: $f(x) = \dfrac{3x^2 + 6x - 12}{x^2}$.

a) Determine the horizontal asymptote(s) of f or tell why it does not have any.

b) Determine the vertical asymptote(s) of f and the left hand and right hand limits of at each vertical asymptote.

c) Find $f'(x)$ (the first derivative of f).

d) Determine the intervals where f is decreasing or increasing; find any local maxima and minima of f.

e) The second derivative of f is given by $f''(x) = \dfrac{12(x-6)}{x^4}$. (Do NOT verify this. You will run out of time if you do.) Determine the intervals where f is concave up or down, and, find any inflection points of f.

f) Using the ABOVE, sketch the graph of f showing and LABELING all of its interesting features. NO credit will be given for only copying a picture from your graphing calculator.

8. (9 points) Use the **definition** of the derivative to find the derivative of $f(x) = 3x^2 + 4$ as a function of x.

9. (8 points) Find the derivative of

$$f(x) = \int_0^{x^3} e^{\cos t}\, dt.$$

10. (11 points each) Evaluate the following indefinite integrals:

a) $\displaystyle\int \frac{x+2}{(x^2+4x+1)^2}\,dx$

b) $\displaystyle\int \frac{\sin x}{2+\cos x}\,dx$

11. (11 points) Evaluate the definite integral: $\displaystyle\int_0^1 x \sin \pi x^2\,dx$