# MAT 397 — SPRING 2005 — EXAM III REVIEW

**Note:** This is not meant to be a comprehensive review. It is intended to remind you the typical sorts of problems that we've considered. I've given some below, and in places where your textbook has decent problems, I've just pointed at them. There are many others on the syllabus.

Exam III covers two general topics: Optimization of multivariate functions, and Integration in two variables. (Observe that there will be no triple integrals on this exam.)

§15.8 Maxima and Minima

You should be able to classify all critical points of a function, whether on all of the $xy$-plane or on a closed bounded set. It's very important to state your answer clearly and carefully, something like "$f(x, y)$ attains its maximum value at $(3, -\pi)$, and that maximum value is $\sqrt{2}$."

(1) Find and classify the critical points of $f(x, y) = x^3y + 12x^3 - 8y$, $g(x, y) = xy(1 - x - y)$, and $h(x, y) = x \sin y$.

(2) Find the global maximum value and global minimum value of $f(x, y) = x^2 + y^2$ on the rectangle $S = \{(x, y) : -1 \leq x \leq 3, -1 \leq y \leq 1\}$.

(3) (the dreaded distance problem) Find the point on the plane defined by $x+y-z = 1$ that is closest to the point $(2, 1, -1)$. What is the distance?

§15.9 Lagrange's Method

These can get hairy. Be sure you understand several ways of solving the system of equations that appears.

(1) Find the extreme values of the function $f(x, y) = x^2+2y^2$ on the circle $x^2+y^2 = 1$. (This can be done without Lagrange multipliers – you should do it both ways to be sure you get the same answer.)

(2) Same question, $f(x, y) = y^2 - x^2$ on the ellipse $x^2 + 4y^2 = 4$.

§16.1 Double Integrals over Rectangles

This is mostly a conceptual section, without many good problems. You should look back at the two problems assigned from this section, though.

§16.2 Iterated Integrals

This is where things get good. Some fine problems on computing iterated integrals are exercises 1–16, 21, 22. Let me know if you want more.

§16.3 Nonrectangular Regions

The most important thing in this section is being fluent in translating (a) descriptions of regions in the plane, (b) sketches of those regions, and (c) limits of integration that cover those regions. Problems 31–36 are good for this. Of course, computing the things is important too, so you should be able to work problems like 1-12.

(1) Evaluate $\iint_D xy\, dA$ where $D$ is the region bounded by the line $y = x - 1$ and the parabola $y^2 = x + 1$.

§16.4 Polar Integrals

This section is again mostly computational. The most important thing is not to forget the $r$ when translating an integral from Cartesian coordinates to polar coordinates. Problems 11–18 are classic.

§16.5 Applications

You need to memorize the formulas for mass of a lamina and the coordinates of its center of mass. Don't worry about the rest of this section (moment of inertia, radius of gyration, etc.). Problems 2 and 3 were assigned for homework.

§16.6 Surface Area

You also need to know the formula for the area of a surface over a region in the plane.

(1) Find the area of the part of the plane $2x + 5y + 7z = 70$ lying over the square $S = \{(x, y) \ : \ -1 \leq x \leq 1, -1 \leq y \leq 1\}$.
(2) Find the area of the part of the saddle $z = x^2 - y^2$ inside the cylinder $x^2 + y^2 = 25$.