MAT 397 — SPRING 2005 — EXAM I REVIEW

Note: This is not meant to be a comprehensive review. It is intended to remind you the sorts of things we've worked on, and to give you a chance to ask questions about typical problems.

(1) Identify each of the following as a parabola, an ellipse, or a hyperbola. Draw a rough sketch (labelling a few points) of the graph of each equation.
  (a) $x^2 + \frac{y^2}{9} = 1$
  (b) $x = -5y^2$
  (c) $x^2 - \frac{y^2}{4} = 1$
  (d) $\frac{x^2}{16} + \frac{y^2}{4} = 1$
  (e) $\frac{y^2}{25} - \frac{x^2}{16} = 1$

(2) Find the symmetries of the curve defined in polar coordinates by $r = 5\sin 2\theta$. Be sure to show all your work.

(3) Find the symmetries of the curve $r = 1 + 2\cos\theta$ and sketch it, labelling at least four points.

(4) Consider the parametrized curve in the plane given by

$$x = \sqrt{2-t}, \qquad y = \sqrt{4t-4}, \qquad 1 \leq t \leq 2\,.$$

  (a) Eliminate t to find the Cartesian equation of this curve. Sketch the graph (labelling some points, as always).
  (b) Find $\frac{dy}{dx}$ at $t = 3/2$. Note that you don't need (a) for this.
  (c) Set up an integral that gives the length of this curve for $1 \leq t \leq 2$. Do not evaluate the integral.

(5) Find the area inside the curve $r = 4\cos\theta$ by computing a polar integral.

(6) Suppose that a small creepy-crawly is moving in the plane according to

$$\vec{r}(t) = (4t+3)\vec{i} + 3t^2\vec{j}\,.$$

  (a) Find $\vec{v}(t)$
  (b) Find $\vec{a}(t)$
  (c) Find $\vec{T}(t)$ (note: it won't simplify much)
  (d) Find $\vec{N}(t)$

(7) Assume that $\vec{v}(t) = 5\sec^2 t\vec{i} + 6e^{2t}\vec{j}$, and that $\vec{r}(0) = 3\vec{i} - 4\vec{j}$. Find $\vec{r}(t)$.

(8) Let

$$\vec{u} = 3\vec{i} + 2\vec{j} + \vec{k}, \quad \vec{v} = 2\vec{i} - 2\vec{j} + 3\vec{k}\,.$$

Find the cosine of the angle between $\vec{u}$ and $\vec{v}$. Compute $\vec{u} \times \vec{v}$.

(9) Find an equation of the plane containing the three points $(1,1,3), (2,1,-1)$, and $(0,4,5)$.

(10) Find parametric and symmetric equations for the line passing through the points $(0,3,2)$ and $(2,-1,5)$.