

## Systems of Equations on the TI-83: Gauss-Jordan Method

We illustrate with the following system of three equations in three unknowns.

$$\begin{aligned}2x + 4y - 3z &= 5 \\3x - 2y + z &= 15 \\x - y + z &= 20\end{aligned}$$

First, we form from this system the following *augmented coefficient matrix*.

$$\begin{bmatrix} 2 & 4 & -3 & 5 \\ 3 & -2 & 1 & 15 \\ 1 & -1 & 1 & 20 \end{bmatrix}$$

The first column consists of the coefficients of  $x$ . The second is the coefficients of  $y$ . The third is the coefficients of  $z$ . The last column contains the constants from the right side of the equation. Each column must consist of the coefficients of one variable only. If a variable does not appear in an equation, its coefficient is zero. The Ti-83 can reduce this matrix (in most cases) to the following *row reduced echelon form*.

$$\begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{bmatrix}$$

If we convert this to a system of equations, we get

$$\begin{aligned}1 \cdot x + 0 \cdot y + 0 \cdot z &= a \\0 \cdot x + 1 \cdot y + 0 \cdot z &= b \\0 \cdot x + 0 \cdot y + 1 \cdot z &= c\end{aligned}$$

Hence,  $x = a, y = b, z = c$ . To do the reduction, perform the following steps on the TI-83:

1. Enter the matrix into the calculator:

Press  $\boxed{2\text{nd}} \overset{\text{matrix}}{\boxed{x^{-1}}}$  to get to the matrix menu. Use the right arrow key to move to the EDIT submenu, then press  $\boxed{\text{ENTER}}$ . From the EDIT menu, select one of the available matrices, e.g. [A] by highlighting it and pressing  $\boxed{\text{ENTER}}$ .

```

NAMES MATH EDIT
1: [A] 3x4
2: [B] 3x4
3: [C] 1x1
4: [D]
5: [E]
6: [F]
7↓ [G]

```

Figure 1: Matrix Menu Names Sub-menu

```

NAMES MATH EDIT
1: [A] 3x4
2: [B] 3x4
3: [C] 1x1
4: [D]
5: [E]
6: [F]
7↓ [G]

```

Figure 2: Matrix Menu Edit Sub-menu

The first two numbers you will need to enter are the number of rows and the number of columns of the matrix. For our example, this is 3 rows and 4 columns. Type the number and then press **ENTER** for each. Next enter each of the numbers in the matrix, starting with the 1st row, 1st column, and proceeding along the 1st row. The cursor will automatically advance to the next position when you press **ENTER**. Use the gray/white **(-)**, not the blue subtract key to enter negative numbers.

```

MATRIX[A] ■ ×1
[ 0

```

Figure 3: Unedited Matrix

```

MATRIX[A] 3 ×4
[ 2      4      -3      -
[ 3      -2      1      -
[ 1      -1      1      -

```

Figure 4: Edited Matrix

When you finish, take another look. The number one problem by far with this method is entering the wrong number. Notice that the calculator cannot display the entire matrix at once. Use the cursor keys to scroll back and forth. Once you are satisfied that you have the matrix

entered correctly, press **2nd** **MODE** to get back to the home screen. If you attempt to do the next step while still in the Matrix menu, you will get an error.

- From the home screen, press  $\boxed{2\text{nd}} \boxed{x^{-1}}$  to return to the matrix menu. Use the right cursor key to go to the MATH submenu. Use the down cursor key to find the rref( function. (This stands for row reduced echelon form. Note that there is also an ref( function. Make sure you choose the one with two r's.) Press  $\boxed{\text{ENTER}}$  to select the function. This returns you to the home screen and puts the rref( function there.

```

NAMES MATH EDIT
1:det(
2:T
3:dim(
4:Fill(
5:identity(
6:randM(
7:augment(

```

Figure 5: MATH Submenu

```

NAMES MATH EDIT
6:randM(
7:augment(
8:Matr▶list(
9>List▶matr(
0:cumSum(
A:ref(
B:rref(

```

Figure 6: rref( function

- From the home screen, press  $\boxed{2\text{nd}} \boxed{x^{-1}}$  to return to the matrix menu one last time. From the names menu, select the matrix which you edited by highlighting it and pressing  $\boxed{\text{ENTER}}$ . This returns you to the home screen, where you should see rref([A]. Close the parentheses and press  $\boxed{\text{ENTER}}$ . The reduced matrix will appear on the screen. You can use the  $\boxed{\text{MATH}} \rightarrow \boxed{\text{MATH}} \rightarrow \text{frac}$  function to convert to fractions, which may make it easier to read.

```

rref([A])
[[1 0 0 8.57142...
 [0 1 0 22.1428...
 [0 0 1 33.5714...

```

Figure 7: Reduced Matrix

```

[[1 0 0 8.57142...
 [0 1 0 22.1428...
 [0 0 1 33.5714...
Ans▶Frac
[[1 0 0 60/7 ]
 [0 1 0 155/7]
 [0 0 1 235/7]]

```

Figure 8: Converted to fractions

4. Convert the reduced matrix back to a system of equations as previously described. For our example, the resulting system is

$$\begin{aligned}x &= \frac{60}{7} = 8\frac{4}{7} \\y &= \frac{155}{7} = 22\frac{1}{7} \\z &= \frac{235}{7} = 33\frac{4}{7}\end{aligned}$$

Do *not* assume that the last column is the values of  $x$ ,  $y$ , and  $z$ . While this is often the case, it will not be true if the system is inconsistent or dependent. If you translate the matrix back into a system of equations, you should be able to make sense of it.

Note: These instructions are for the TI-83 plus and TI-83 silver edition. The standard TI-83 (non-plus) has a `MATRIX` key instead of a `2nd` function, otherwise everything works the same. The TI-82 des not have an `rref(` function. A program to do Gauss-Jordan method is available from me. For other TI, HP, Casio, etc graphing calculators, I can only hope that you kept your manual.