This exam has 7 problems on 8 pages. Name: ______________________

Show all work for full credit. Score: ________ /100

Put a box around your answer.

1. (12 pts) The following data is gathered by an experimenter.

x_1	0	1	−1	1
x_2	2	1	1	−2
y	2	0	2	−3

Find the linear function $y = ax_1 + bx_2$ that best fits this data.

2. (24 pts) Find the $1-$, $2-$, and $\infty-$ norms of the following vectors.

(a) $(0,\ -1,\ 2,\ -2,\ 4)^T$

(b) $(2,\ -1,\ i,\ 3-2i)^T$

3. **(a)** (9 pts) Find the $1-$, $\infty-$, and Frobenius norms of the matrix.

$$A = \begin{pmatrix} 3 & 0 & -1 \\ 2 & 2 & -2 \\ 0 & 1 & -2 \end{pmatrix}$$

(b) (6 pts) Find the 2-norm of the matrix A.

$$A = \begin{pmatrix} 2 & 0 \\ -1 & \sqrt{3} \end{pmatrix}$$

4. (16 pts) Are the following matrices symmetric, hermitian, orthogonal, unitary? (Answer yes or no for each property.)

(a) $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ -1 & 0 & 1 \end{pmatrix}$

(b) $\frac{1}{\sqrt{3}}\begin{pmatrix} 1+i & i \\ i & -i \end{pmatrix}$

5. Consider the following orthogonal vectors in $\mathbb{C}^3$.

$$u_1 = \begin{pmatrix} 1 \\ i \\ 1 \end{pmatrix}, \qquad u_2 = \begin{pmatrix} i \\ 1 \\ 0 \end{pmatrix}$$

(a) (5 pts) Find a third vector u_3 such that $\{u_1,\ u_2,\ u_3\}$ is an orthogonal basis.

(b) (6 pts) Write $w = (1,\ 1,\ 1)^T$ as a linear combination of $u_1,\ u_2,\ u_3$.

6. (12 pts) Find a QR decomposition for the matrix. (You may use either the “classical” or “modified” approach.)

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & -1 & 2 \\ 1 & -1 & 2 \\ -1 & 1 & 1 \end{pmatrix}$$

7. (10 pts) Let $u = (3,\ 0,\ 4)^T$.

(a) Write down the elementary reflector corresponding to u.

(b) Find the projection of $(2,\ -1,\ 4)^T$ onto $u^{\perp}$.