MAT 532 — HOMEWORK 5

DUE ON THURSDAY 4 OCTOBER

1. A matrix A is called *symmetric* if $A^T = A$, *skew-symmetric* if $A^T = -A$, *Hermitian* if $A^* = A$, and *skew-Hermitian* if $A^* = -A$. Is $\begin{bmatrix} 1 & 2+4i & 1-3i \\ 2-4i & 3 & 8+6i \\ 1+3i & 8-6i & 5 \end{bmatrix}$ symmetric, skew-symmetric, Hermitian, or skew-Hermitian?

2. For each of the following pairs consisting of a matrix A and its inverse, find $\|A\|$, $\|A^{-1}\|$ and $\kappa(A)$, all with respect to the ∞-norm $\|\cdot\|_\infty$ (AKA the maximum absolute row sum).

(a)

$$A = \begin{bmatrix} 1000 & 999 \\ 999 & 998 \end{bmatrix} \qquad \text{and} \qquad A^{-1} = \begin{bmatrix} -998 & 999 \\ 999 & -1000 \end{bmatrix}$$

(b) (the Hilbert matrix)

$$H_5 = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{bmatrix}, \quad H_5^{-1} = \begin{bmatrix} 25 & -300 & 1050 & -1400 & 630 \\ -300 & 4800 & -18900 & 26880 & -12600 \\ 1050 & -18900 & 79380 & -117600 & 56700 \\ -1400 & 26880 & -117600 & 179200 & -88200 \\ 630 & -12600 & 56700 & -88200 & 44100 \end{bmatrix}$$

3. Let

$$A = \begin{bmatrix} 2 & 4 & -10 & 6 & 28 \\ 0 & 0 & -2 & 0 & 12 \\ 2 & 4 & -5 & 6 & -1 \end{bmatrix}.$$

(a) Find a matrix P that is a product of elementary matrices of types I, II, III, such that $PA = E_A$ is the reduced row-echelon form of A.

(b) Find matrices P and Q that are both products of elementary matrices of types I, II, III, such that PAQ is in rank normal form. (Recall that this means PAQ looks like an $r \times r$ identity matrix in the upper left and 0s everywhere else.)

4. Let

$$A = \begin{bmatrix} 3 & 6 & -9 \\ 2 & 5 & -3 \\ -4 & 1 & 10 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 24 \\ 23 \\ 26 \end{bmatrix}.$$

(a) Find an LU decomposition for A.

(b) Use your decomposition to solve $Ax = b$.

(c) Find an LDU decomposition for A.

5. Let

$$A = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 3 & 3 & 1 & 4 \\ 7 & 9 & 5 & 8 \\ 7 & 9 & 8 & 6 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -2 \\ -4 \\ -10 \\ -12 \end{bmatrix}.$$

(a) Explain why A does not have an LU decomposition.

(b) Find a permutation matrix P and an LU decomposition $PA = LU$.

(c) Solve $Ax = b$ using your decomposition.