

**MAT 631 — HOMEWORK 9**

DUE ON TUESDAY 6 NOVEMBER

1. let  $Q_8$  be the quaternion group of order 8.
  - (a) Prove that  $Q_8$  is isomorphic to a subgroup of  $S_8$ .
  - (b) Prove that  $Q_8$  is *not* isomorphic to a subgroup of  $S_n$  for any  $n \leq 7$ . (Hint: prove that in any action of  $Q_8$  on a set of size  $\leq 7$ , the stabilizer of any point must contain the unique element of order 2, namely  $-1$ .)
2. Prove that conjugate elements of a group have the same order. Prove that conjugate subgroups have the same order.
3. Find all conjugacy classes and their sizes for:
  - (a)  $A_4$
  - (b)  $S_3 \times S_3$
4. If the center of a group has index  $n$ , prove that every conjugacy class has size at most  $n$ .
5. List all partitions of 6 and give representatives for the corresponding conjugacy classes of  $S_6$ .