

MAT 534 — HOMEWORK 6

DUE ON FRIDAY 28 FEBRUARY

All these problems are from Chapter 7.

1. (#1) Let H be the subgroup of A_4 consisting of the elements $\{(1), (12)(34), (13)(24), (14)(23)\}$. (You don't have to check that this is a subgroup, but it's a good idea to do it for yourself. Also convince yourself that H is indeed contained in A_4 .) Find all the left cosets of H in A_4 , using the table on page 111.
2. (#12) Let a and b be non-identity elements of a group of order $155 = 5 \times 31$, and assume that $|a| \neq |b|$. Prove that the only subgroup of G containing both a and b is G itself.
3. (#15) What are the possible orders of subgroups of a group of order 60? Justify.
4. (#19) Compute $5^{15} \bmod 17$ and $7^{13} \bmod 11$. (Be smart – use Fermat's Little Theorem.)
5. (#22) Suppose H and K are subgroups of some group G , and that $|H| = 12$ and $|K| = 35$. Find $|H \cap K|$.
6. (#34) Suppose that a group contains elements of orders $1, 2, \dots, 10$. What is the minimal possible order of the group?
7. (#45) Let $G = \{(1), (12)(34), (1234)(56), (13)(24), (1432)(56), (56)(13), (14)(23), (24)(56)\}$. Find the orbits of 1 and 5, and the stabilizers of 1 and 5.