MAT 534 — HOMEWORK 7

DUE ON FRIDAY 7 MARCH

All these problems are from Chapter 8.

1. (#1) Prove that the external direct product of any finite number of groups is a group.

2. (#4) Show that $G \oplus H$ is Abelian if and only if both G and H are Abelian.

3. (#5) Is $\mathbb{Z} \oplus \mathbb{Z}$ cyclic? Justify.

4. (#6) Prove that $\mathbb{Z}_2 \oplus \mathbb{Z}_8$ is *not* isomorphic to $\mathbb{Z}_4 \oplus \mathbb{Z}_4$, by comparing orders of elements.

5. (#7) Prove that $G \oplus H \cong H \oplus G$. (You should write down an explicit isomorphism.)

6. (#13) For each integer $n > 1$, give examples of two non-isomorphic groups of order n^2. Justify.

7. (#14) The dihedral group D_n of order $2n$ ($n \geq 3$) has a subgroup of n rotations, and a subgroup of order 2. (In fact there are several subgroups of order 2 – pick one.) Explain why D_n is not isomorphic to the external direct product of two such subgroups.

Also suggested, but not to hand in:

(A) (#17) If $G \oplus H$ is cyclic, prove that G and H are cyclic. Is the converse true?

(B) (#3, improved) Let G and G' be groups, and let $H \leq G$ and $H' \leq G'$ be subgroups. Prove that $H \oplus H'$ is isomorphic to a subgroup of $G \oplus G'$. In the special case $H' = \{e'\}$ is the trivial subgroup of G', prove that $H \cong H \oplus \{e'\}$.

(C) (#28) Find a subgroup of $\mathbb{Z}_4 \oplus \mathbb{Z}_2$ that is not of the form $H \oplus K$ for two subgroups $H \leq \mathbb{Z}_4$ and $K \leq \mathbb{Z}_2$.