MAT 534 — HOMEWORK 11

DUE ON WEDNESDAY 16 APRIL

1. (Ch. 12, #19) Let R be a ring. The *center* of R is the set $\{x \in R \mid ax = xa \text{ for all } a \in R\}$. Prove that the center of R is a commutative subring.

2. (Ch. 12, #22) Let R be a commutative ring with identity and let $U(R)$ denote the set of units of R. Prove that $U(R)$ is a group under the multiplication of R.

3. (Ch. 13, #4) List the zero-divisors of $\mathbb{Z}_{20}$. Can you see a relationship between the zero-divisors of $\mathbb{Z}_{20}$ and the units of $\mathbb{Z}_{20}$?

4. (Ch. 13, #11 and #30) Let d be a positive integer.

(a) Prove that $\mathbb{Z}[\sqrt{d}] := \left\{a + b\sqrt{d} \;\middle|\; a, b \in \mathbb{Z}\right\}$ is an integral domain.

(b) Prove that $\mathbb{Q}[\sqrt{d}] := \left\{a + b\sqrt{d} \;\middle|\; a, b \in \mathbb{Q}\right\}$ is a field.

5. (Ch. 13, #15) Let a be an element of a ring R with identity, and suppose that $a^n = 0$ for some n. (Such an element is called *nilpotent*.) Prove that $1 - a$ has a multiplicative inverse in R. (Hint: consider the geometric series $\sum_k a^k = \frac{1}{1-a}$.)